Detecting Wheel Rotation

R H Brown 24 September 2002

Summary: a simple mathematical description of a rotating wheel has been created to allow the acceleration experienced at the valve stem location to be modelled. The effect of AC-coupling this acceleration signal (as would result from the use of a piezoelectric accelerometer) can then be investigated. A discrete-time model has been used, which allows the effect of the high-pass filter to be applied using FFT routine. The results show that a high-pass filter with cut-off frequency at around 1 Hz should be sufficient to detect the onset of rotation (within a few rotations), and that a higher-order filter (greater than single-order RC) may be advantageous in some circumstances.

A complete listing of the MathCad program is attached at the end of this document.

Introduction

There are two components of acceleration present in a rotating wheel: the oscillating signal due to the relative orientation of the accelerometer to gravitational pull (+/- 1 g), and the centripetal acceleration given by v^2/R. If a DC-coupled accelerometer technology (e.g. capacitive, piezoresistive) is used as a detector, the rotational (centripetal) component will very quickly dominate, with the gravitational component appearing only as a slight ripple superimposed on the much slower, but higher magnitude, centripetal signal.

To detect the onset of wheel rotation, a DC-coupled device could be used to detect when the centripetal acceleration exceeds a given threshold (implying a fixed vehicle speed threshold).

If an AC-coupled (e.g. piezoelectric) accelerometer is used, however, the majority of the centripetal acceleration signal may be lost (depending upon the exact location of the –3 dB cut-off frequency). The purpose of this model is to investigate the approximate conditions which could result in a threshold-detection of rotation.

Setting up the model

The attached MathCad listing annotates each step, but basicaly four time periods are included: an initial period with no motion, an acceleration phase from rest to a maximum speed, a constant speed phase, and a deceleration phase back to rest. The duration of each phase can be varied, and if the sum of all periods is less than the total time record, a further phase of zero speed is assumed.

Both the acceleration and deceleration phases are assumed to be linear. This simplifies the model, and any practical level of acceleration can be investigated. The shape of the acceleration profile itself is not considered to be particularly significant in this analysis.

The tire circumference can be altered, as can the radius at the valve stem, to suit any given wheel geometry. Flattening of the tire is not taken into account in the "distance travelled" calculations – again, this is not considered essential to the investigation.

Four plots are generated initially: vehicle speed versus time, distance travelled versus time, gravitational acceleration versus time, and total acceleration (centripetal plus gravitational) versus time.

Applying the high-pass filter

In the final section of the model, a simple high-pass filter is created by modelling the effect of an R-C combination (resistance & capacitance). This is particularly appropriate, since any piezoelectric sensor will have a fixed capacitance, and will most likely see a fixed value of input resistance within the electronic interface. By selecting appropriate value of C (equivalent to typical value for small area & thickness of piezo film), and anticipated value of R, the model directly predicts the effect on the final output signal.

The influence of this RC combination is computed for each frequency line of the spectrum of the acceleration signal, and the product of the acceleration and filter spectra is taken. Finally, inverse FFT is used to return to the time domain.

The resulting plot is thus the acceleration signal as seen after passing through the high-pass filter. In further analyses, the HPF characteristic was "squared" or rasied to even higher powers (3^{rd} or 4^{th}) before applying to the acceleration signal. This is equivalent to the effect of a 2^{nd}, 3^{rd} or 4^{th} order HPF with same cut-off frequency.

Observations

With the centripetal component of acceleration largely removed, a single-order HP-filtered signal shows the +/- 1g gravitation signal very clearly. Under steady acceleration, a slight linear increase can still be seen, which disappears at constant speed. Fast deceleration causes a significant negative-going "transient", where the gravitational signal would be virtually lost.

By setting other values within the model, it has been found that an approximate +0.5 g threshold would be appropriate to detect onset of rotation, even when the acceleration rate is very low (e.g. from rest to 5 kph in 5 seconds) – trigger occurs after either one or two complete rotations (after one, threshold is just crossed).

Using higher-order HP characteristic, the linear increase at fast acceleration is almost completely removed, and the negative-going transient under fast braking is reduced in amplitude and duration. A 2^{nd} order filter gives good response, with 3^{rd} or higher orders not improving the signal significantly.

Whether the 2^{nd} order response is actually required in practice would depend upon the algorithm used to detect (and maintain "condition" of) rotation.

Alternatively, a higher cut-off frequency of 1^{st} order HPF could be used (to remove more of the centripetal signal), although this would increase the distance or speed required to trigger detection. A 1 Hz cut-off frequency appears to be approximately best for detecting the gravitational signal.

With around 0.5 g trigger level, an accelerometer with 10 mV/g sensitivity (such as MSI's ACH-01) would give only 5 mV trigger level, so may require gain of 100 or so to bring signals up to usable level. Alternative components such as LDTC with around 1 V/g sensitivity at low frequency may also be suitable, although in this case the mass-loaded cantilever structure would be expected to "bottom out" at high centripetal acceleration. Signal may be lost entirely, until speed reduces to lower levels. Again, this may still be acceptable, depending upon the detection algorithm.

WHEEL6.MCD R H Brown MSI Sensor Products Division 27 March 2001

This program calculates gravitational and rotational acceleration for typical wheel at varying road speeds. It also allows a high-pass filter to be applied to the "ideal" acceleration signal (using FFT). A discrete-time (sampled data) approach is used.

$N := 4096$ # time samples, must be integer power of 2 (for FFT)
(Mathcad v 3.1 will accept max 4096 here)

$i := 0, 1 .. N - 1$ $k := 0, 1 .. \frac{N}{2}$ (range variables)

$samptime := 0.002$ sampling time interval, in seconds

$time_i := i \cdot samptime$ $sampfreq := \frac{1}{samptime}$

$freq_k := \frac{k}{N} \cdot sampfreq$

$sampfreq = 500$

$maxtime := (N - 1) \cdot samptime$

$maxtime = 8.19$ overall length of time record

$t0 := 0$ t0 is the "pre-trigger delay" prior to moving off

$t1 := 5$ $maxvel := 50$ t1 is time to accelerate from 0 to full speed
maxvel is full speed, in kph

$pos := \frac{maxvel}{t1}$ $pos = 10$ pos is constant positive acceleration
typical practical max = 20 (0 to 100 kph in 5 s)

$t2 := 0.5$ $cvel := maxvel$ t2 is dwell time at max speed

$t3 := 1.002$ $minvel := 0$ t3 is time to decelerate back to 0. This can be adjusted to leave wheel in correct "phase" for later FFT

$neg := \frac{maxvel}{t3}$ $neg = 49.9$ neg is constant deceleration value
typical practical min = 40?

$cyctime := t0 + t1 + t2 + t3$

$cyctime = 6.502$ check that total cycle time fits within the time record (cyctime < maxtime)!

Define the speed vs time relationships:

$c1 := t0$ $c2 := c1 + t1$ $c3 := c2 + t2$ $c4 := c3 + t3$

$velk_i := if\left[\left(time_i < c2\right) \cdot \left(time_i \geq t0\right), \left(time_i - c1\right) \cdot pos, 0 \right]$

$velk_i := if\left(time_i \geq c2, maxvel, velk_i\right)$

$velk_i := if\left[time_i \geq c3, \left(c4 - time_i\right) \cdot neg, velk_i \right]$

$velk_i := if\left(time_i \geq c4, 0, velk_i\right)$

$velk_i := if\left(time_i < t0, 0, velk_i\right)$

$velm_i := \frac{velk_i}{3.6}$ convert speed in kph to m/s

$dist_0 := 0$ define initial distance travelled as zero, then form running integral for each successive time increment

$dist_i := (i>1)\cdot\left(dist_{i-1} + velm_i \cdot samptime\right)$

$circ := 1.621$ define circumference of typical tire (Windstar), in m

$mant(x) := x - floor(x)$ a function to extract fractional portion of distance divided by circumference

$distrem_i := mant\left(\frac{dist_i}{circ}\right)$

$grav1_i := \sin\left(distrem_i \cdot 2\cdot\pi\right)$ calculates the gravitational acceleration "seen" by the sensor, according to the position of the wheel. Only grav1 is used in following analysis, but any "phase" of initial starting point is possible.

$grav2_i := -\sin\left(distrem_i \cdot 2\cdot\pi\right)$

$grav3_i := \cos\left(distrem_i \cdot 2\cdot\pi\right)$

$grav4_i := -\cos\left(distrem_i \cdot 2\cdot\pi\right)$

$rps_i := \frac{velm_i}{circ}$ calculates the revs per sec (Hz), and displays max value

$max(rps) = 8.568$

$radfreq_i := rps_i \cdot 2\cdot\pi$ converts revs/s into radian frequency (rad/s)

$radius := 0.208$ Note: radius at valve stem location (not outer tire radius)

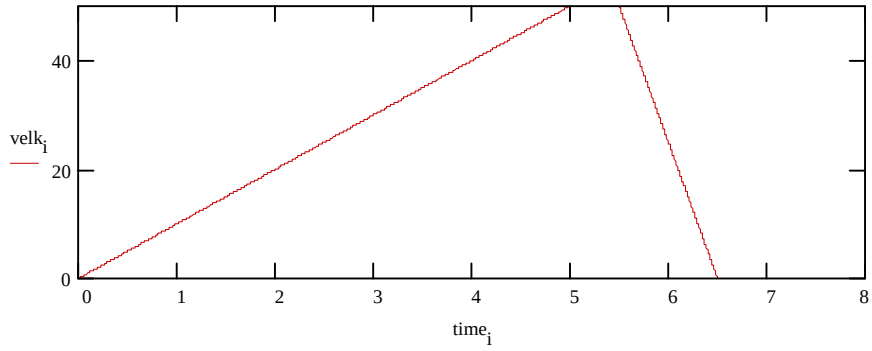
$accm_i := \left(radfreq_i\right)^2 \cdot radius$ calculates centripetal acceleration (in m/s/s)

$accg_i := \frac{accm_i}{9.81}$ converts to units of "g"

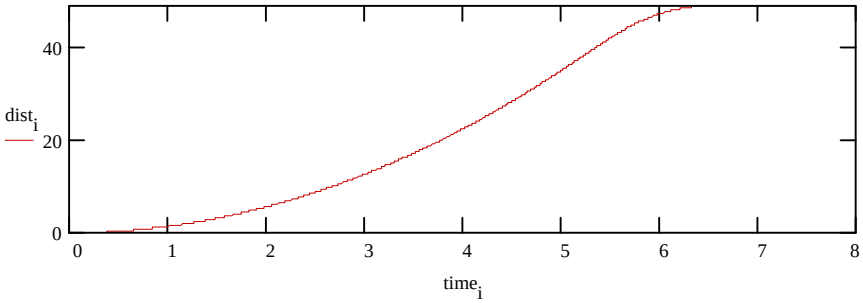
$acc_i := accg_i + grav1_i$ total instantaneous acceleration = sum of gravitational and centripetal

Next, add plots of various parameters versus time (see next page)

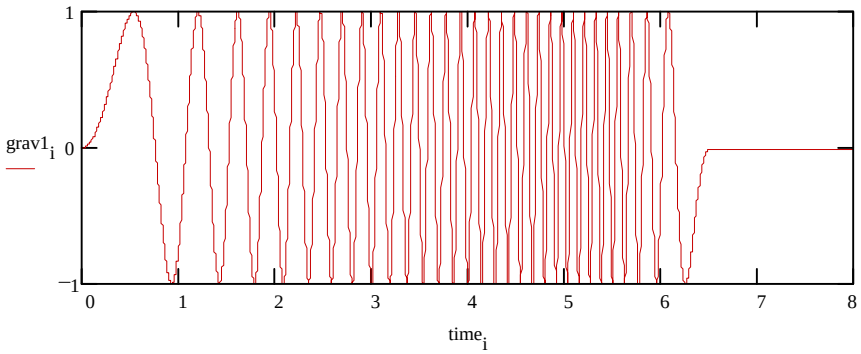
Speed in kph vs time:



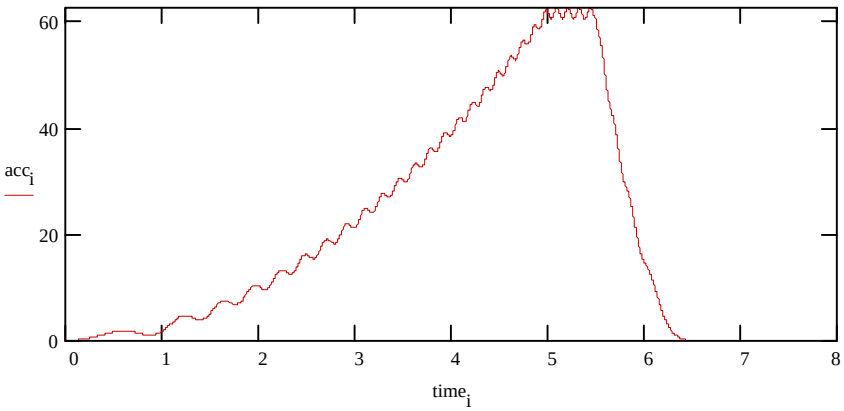
Total distance travelled vs time:



Gravitational acceleration (in g) vs time:



Total acceleration (in g) vs time:



This section applies an R-C high-pass filter to the total acceleration signal:

$mag := fft(acc)$ — calculate spectrum of the acceleration time signal

$C := 5.0 \cdot 10^{-11}$ — set value of source capacitance

$R := 3 \cdot 10^{9}$ — set value of input resistance

$R \cdot C = 0.15$ — display time constant (in s)

$fc := \frac{1}{2 \cdot \pi \cdot R \cdot C}$ — calculate & display cut-off frequency (-3 dB)

$fc = 1.061$

$\omega := 2 \cdot \pi \cdot \overrightarrow{freq}$ — convert frequency values to from Hz to radians

$\omega_0 := 0.00001$ — insert finite (non-zero) freq for DC (required because this term appears in denominator)

$X := \overrightarrow{\frac{1}{\omega \cdot C}}$ — reactance of capacitor over frequency

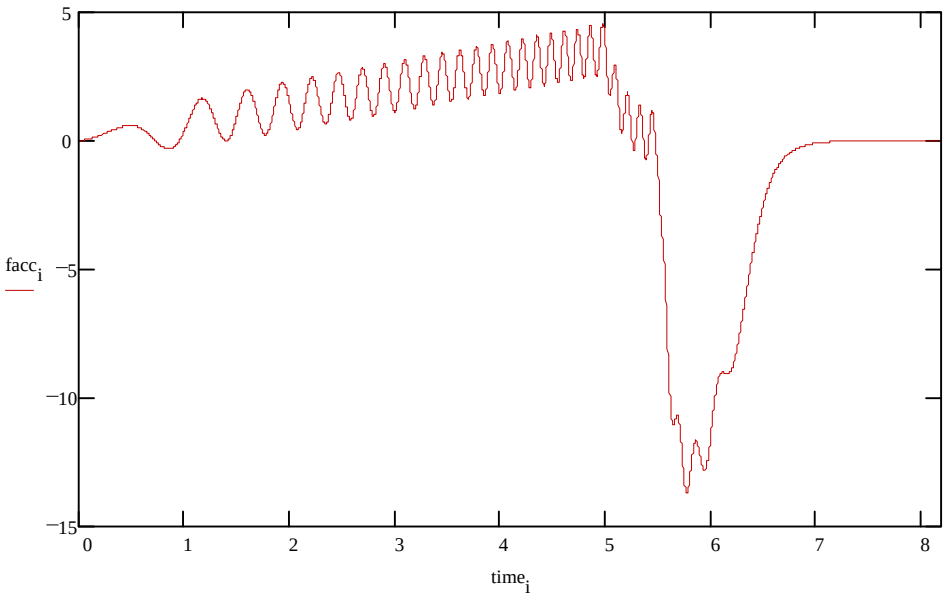
$Z := \overrightarrow{(j \cdot X)}$ — impedance of capacitor

$Vout := \overrightarrow{\frac{R}{R + Z}}$ — calculate output of R-C filter

$out := \overrightarrow{(mag \cdot Vout)}$ — apply RC filter to the acceleration spectrum (note: can change exponent of Vout to yield higher-order filter)

$facc := ifft(out)$ — finally, take inverse FFT to convert filtered spectrum back into time domain

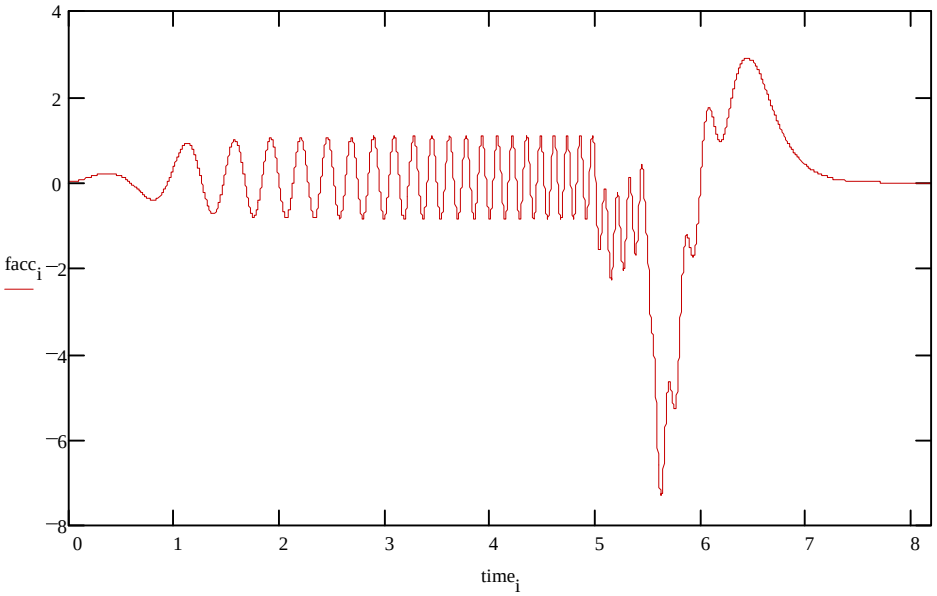
Acceleration signal after high-pass filter:



$out := \overrightarrow{(mag \cdot Vout^2)}$ apply RC filter to the acceleration spectrum (note: can change exponent of Vout to yield higher-order filter)

$facc := ifft(out)$ finally, take inverse FFT to convert filtered spectrum back into time domain

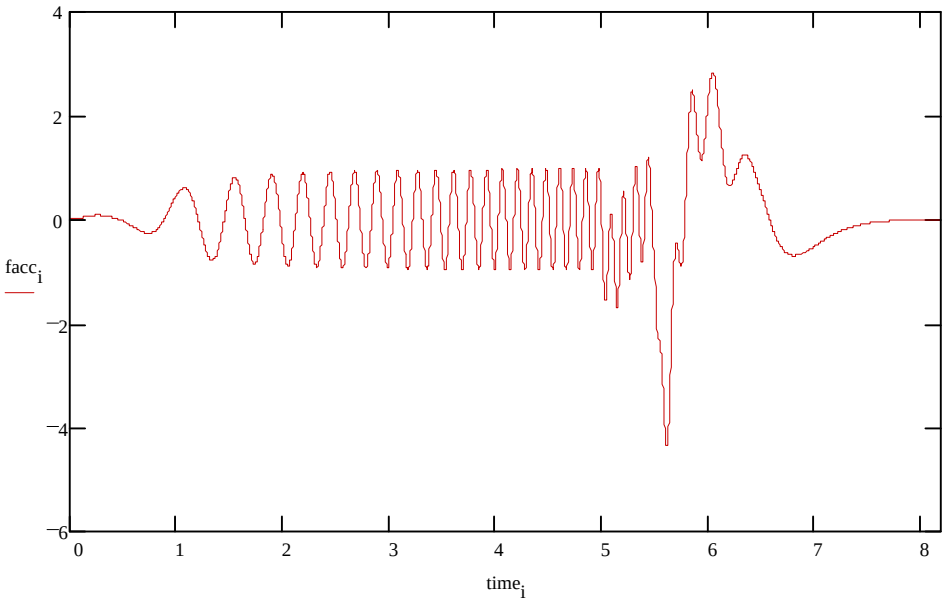
Acceleration signal after high-pass filter:



$out := \overrightarrow{(mag \cdot Vout^3)}$ apply RC filter to the acceleration spectrum (note: can change exponent of Vout to yield higher-order filter)

$facc := ifft(out)$ finally, take inverse FFT to convert filtered spectrum back into time domain

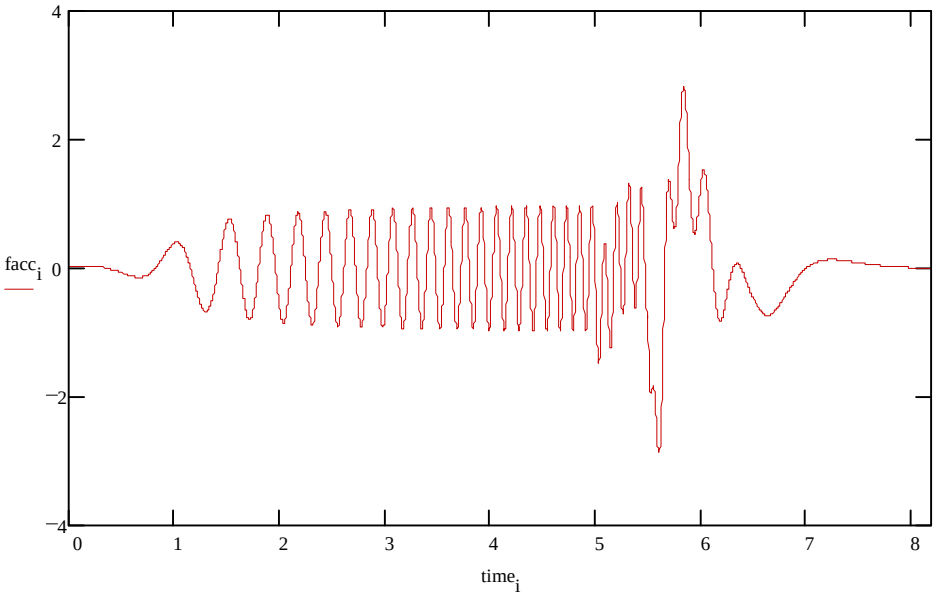
Acceleration signal after high-pass filter:



$out := \overrightarrow{(mag \cdot Vout^4)}$ apply RC filter to the acceleration spectrum (note: can change exponent of Vout to yield higher-order filter)

$facc := ifft(out)$ finally, take inverse FFT to convert filtered spectrum back into time domain

Acceleration signal after high-pass filter:



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